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# Phase Unwrapped Method of Modified Fringe Order

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**Abstract:** Phase measurement profilometry is an optical three-dimensional measurement method, in which phase is the key factor to accurately obtain the three-dimensional coordinates of the measured object, but there will be phase jump error in phase unwrapped process. In order to eliminate the phase jump error existing in the phase unwrapped process based on the principle of multi-frequency heterodyne, a phase unwrapped method with modified fringe orders is proposed. Modifying the integer part of the fringe orders according to the relationship between the integer part of the fringe orders of adjacent pixels can effectively eliminate the jump of the fringe orders caused by the small error, and avoid the transmission and amplification of the phase jump error in the process of fringe heterodyne. Using the multi-frequency characteristic, the absolute phase of the fringe with the middle period is solved by combining the phase information of the fringe with different periods, so as to improve the accuracy of the phase details of the solution. Both simulation and experimental results show that the proposed method has obvious correction effect on the phase jump error, and the unwrapped phase is smoother. Compared with existing methods, the phase error of the proposed method is reduced by an average of 23%.

**Keywords:** Measurement, Structured Light, Three-Frequency Heterodyne, Phase Unwrapped, Phase Error Correction

## 1. Introduction

Structured light three-dimensional detection technology is a three-dimensional space topography detection technology based on digital fringe projection. It is widely used in many fields due to its advantages of non-contact, cheap and convenient equipment, and high detection accuracy [1-4]. The phase shift method is the most common measurement method in the current structured light three-dimensional detection and has a high detection accuracy. However, since the result of using the phase shift method to solve the phase is a wrapped phase with a value range of [-π, π], and the wrapped phase has ambiguous and discontinuous features in the entire detection range, so it is necessary to perform phase unwrapped to obtain a continuous, unambiguous absolute phase over the entire detection range.

In the actual detection process, various unfavorable factors such as environmental noise, the nonlinear response of digital projectors and cameras greatly affect the accuracy of phase unwrapped, resulting in low three-dimensional detection accuracy of structured light and reduced ability to recognize details. The multi-frequency heterodyne principle directly solves the absolute phase, which will generate jump errors [5-7]. In this regard, researchers at home and abroad have proposed various phase error correction methods to suppress the jump error and improve accuracy of phase solution. Phase error correction methods can be summarized into three categories: active correction method, passive correction method and reverse correction method. The active correction method is to re-encode the fringes to be projected, so that the camera can capture more sinusoidal fringe images. For example, Zheng [8] used genetic algorithm to calculate the appropriate the precoding value is introduced into the fringes for re-encoding, but when the system environment and device parameters are changed, the calculation needs to be re-calculated; Zhang [9] adopted the defocus technology of binary fringe, when the projector was in defocus state, the binary image with good encoding was projected to filter out the high order harmonics, thus reducing the phase error. However, the influence of defocus degree and fringe width should be considered. The passive correction method modifies the phase error by calibrating the relationship between the phase error and the phase value. For example, Zhang [10] constructed a phase error look-up table by calibrating the corresponding relationship between the fringe phase value and the phase error, and used the look-up table in the measurement process. Xing [11] used the iterative least squares self-correction algorithm to determine the phase error coefficient using the dual-frequency fringe phase. The passive calibration method is sensitive to changes in the system environment, the calibration process is complicated, and the detection speed and accuracy are restricted by the complexity of the calibration algorithm. The reverse correction method modifies the phase by studying the cause of the phase error. For example, Chen [12] used the relationship between the phase and the fringe period to correct the phase, but this method has many constraints, and because the third fringe period is too large, it does not significantly help to improve the detection accuracy.

Therefore, this paper proposes a phase unwrapped method that modifies the fringe orders to eliminate the jump error existing in the phase unwrapped process based on the multi-frequency heterodyne principle and improve the three-dimensional detection accuracy of structured light. Using the multi-frequency feature of the multi-frequency heterodyne principle, the fringe orders is modified, and the absolute phase is solved by using the fringe phase information of multiple periods to improve the ability of the absolute phase to identify details. After phase unwrapped by the method of this paper, the jump error is eliminated, and the absolute phase is smoother.

# 2. Phase Solution and Error Analysis of Three-Frequency Heterodyne Method

## 2.1. Four-Step Phase Shift Method for Wrapped Phase

The four-step phase shift method [13] is widely used in structured light three-dimensional measurement due to its ability to suppress the influence of higher harmonics in fringe images. When the four-step phase shift method is used to solve the wrapped phase of the fringe image, the phase shift amount of each step is π/2, that is, 4 fringe images with phase shift values of 0, π/2, π and 3π/2 need to be projected and its light intensity distribution is

$$\begin{aligned} I_1(x,y) &= a + b\cos[\varphi(x,y)] \\ I_2(x,y) &= a + b\cos[\varphi(x,y)+\pi/2] \\ I_3(x,y) &= a + b\cos[\varphi(x,y)+\pi] \\ I_4(x,y) &= a + b\cos[\varphi(x,y)+3\pi/2] \end{aligned} \tag{1}$$

By combining the above four equations, the corresponding wrapped phase can be obtained as

$$\varphi(x,y) = \arctan(\frac{I_4 - I_2}{I_1 - I_3}), -\pi \le \varphi \le \pi \tag{2}$$

The wrapped phase obtained at this time is in a zigzag distribution in the range of [-π, π], so it is necessary to perform phase unwrapped to obtain a continuously distributed and unambiguous absolute phase.

## 2.2. The Principle of Three-Frequency Heterodyne Method

The specific process of the three-frequency heterodyne method [14] is shown in Figure 1. The solid line part in the figure is the process of establishing a continuous phase by combining the wrapped phases corresponding to three different periods, while the dotted line part is the process of deriving the fringe orders in reverse using the established continuous phase. $\varphi_i$ represents the wrapped phase corresponding to the fringe with period $T_i$, where $i$ =1, 2, 3, 12, 23. $\varphi_1$ and $\varphi_2$, $\varphi_2$ and $\varphi_3$ heterodyne to obtain the wrapped phases $\varphi_{12}$ and $\varphi_{23}$, then any point on the surface of the measured object satisfies:

$$T_{12} = \frac{T_1 T_2}{T_2 - T_1}, T_{23} = \frac{T_2 T_3}{T_3 - T_2} \tag{3}$$

$$\Delta m_i = \frac{\varphi_i + \pi}{2\pi}, \Delta m_i \in [0,1), i = 1,2,3,12,23 \tag{4}$$

$$\begin{aligned} \Delta m_{12} &= \begin{cases} \Delta m_1 - \Delta m_2, \Delta m_1 - \Delta m_2 \ge 0 \\ \Delta m_1 - \Delta m_2 + 1, \Delta m_1 - \Delta m_2 < 0 \end{cases} \\ \Delta m_{23} &= \begin{cases} \Delta m_2 - \Delta m_3, \Delta m_2 - \Delta m_3 \ge 0 \\ \Delta m_2 - \Delta m_3 + 1, \Delta m_2 - \Delta m_3 < 0 \end{cases} \end{aligned} \tag{5}$$

$$\phi_i = 2\pi \times m_i = 2\pi \times (M_i + \Delta m_i), M_i \in Z, i = 1,2,3,12,23 \tag{6}$$

In the above equation, $T_{12}$ and $T_{23}$ represent the period of the fringe 12 and the fringe 23 respectively, $m_i$ represents the fringe orders of a certain point on the surface of the measured object in the corresponding fringe image, $m_i$ contains an integer part $M_i$ and a fractional part $\Delta m_i$, $\varphi_i$ represents the wrapped phase of the corresponding fringe, $\phi_i$ represents the absolute phase of the corresponding fringe. The fringes 12 and 23 are heterodyne to form fringe 123. By selecting appropriate $T_1$, $T_2$, $T_3$, the measurement range of the fringe 123 can cover the entire viewing angle of the camera, at this time $M_{123} = 0$.

Therefore

$$\Delta m_{123} = \begin{cases} \Delta m_{12} - \Delta m_{23}, \Delta m_{12} - \Delta m_{23} \ge 0 \\ \Delta m_{12} - \Delta m_{23} + 1, \Delta m_{12} - \Delta m_{23} < 0 \end{cases} \tag{7}$$

$$m_{12} = \frac{T_{23}(M_{123} + \Delta m_{123})}{T_{23} - T_{12}}, M_{12} = round(m_{12}) \tag{8}$$

$$\phi_1 = 2\pi \times M_1 + \varphi_1, M_1 = round[\frac{T_2(M_{12} + \Delta m_{12})}{T_2 - T_1}] \tag{9}$$

Where *round* () means rounding.

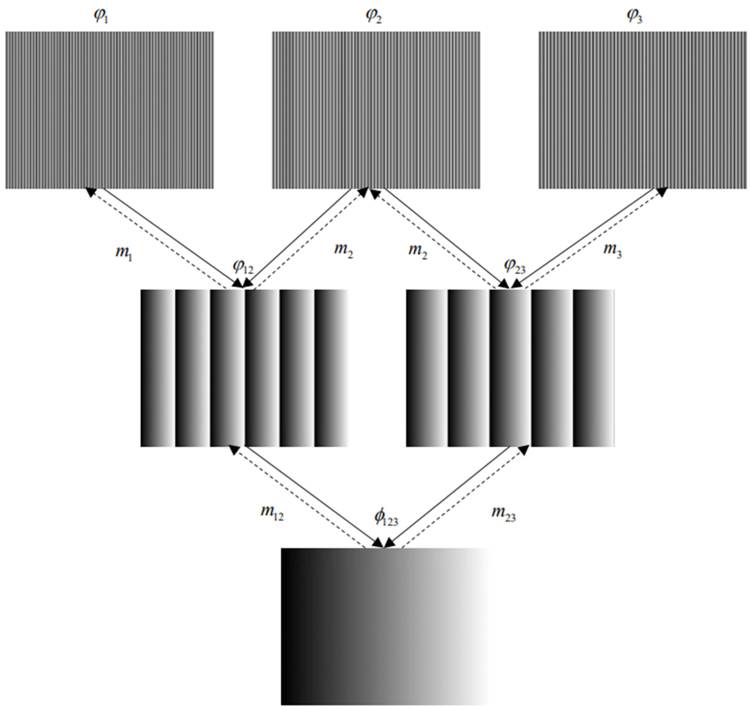
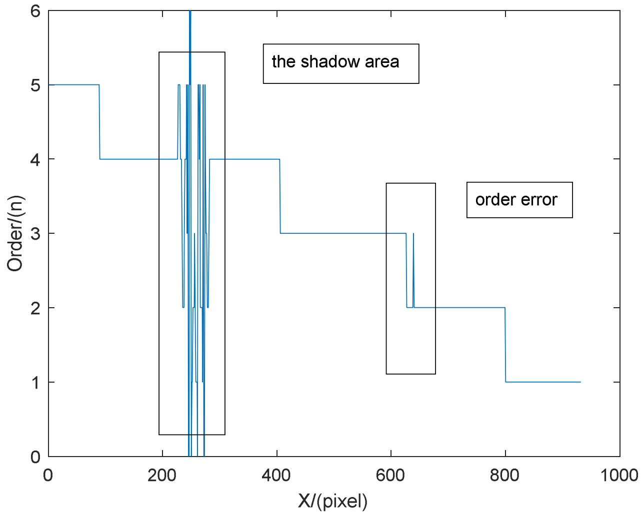
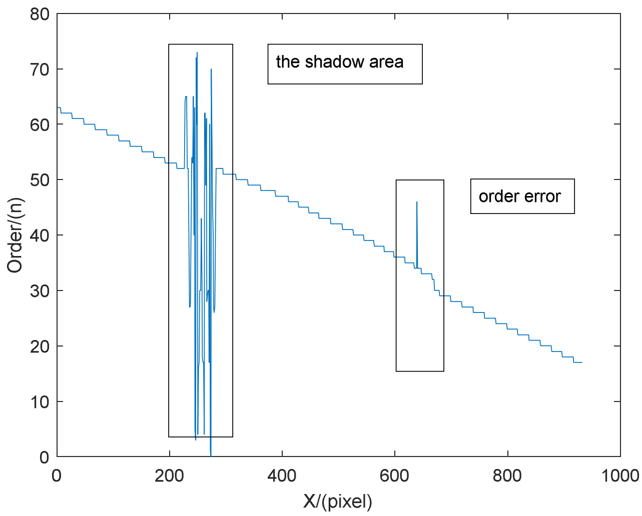


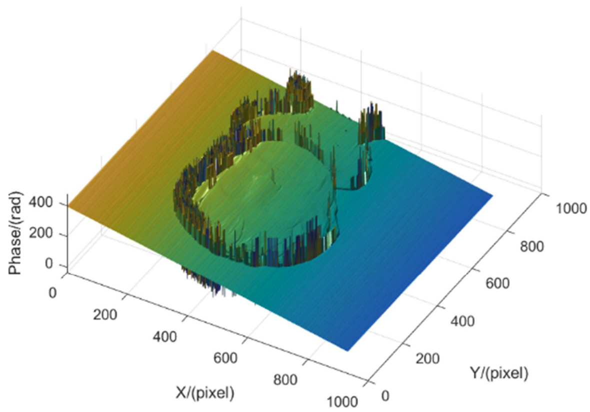

***Figure 1.*** *The process of the three-frequency heterodyne method.*




(a) $M_{12}$




(b) $M_1$




(c) $\phi_1$

***Figure 2.*** *Jump error.*

### 2.3. Phase Unwrapped Error Analysis

Under ideal conditions, the ideal absolute phase can be solved according to the three-frequency heterodyne method described in Section 2.2, but in the actual detection process, the absolute phase obtained by the three-frequency heterodyne method is easily affected by environmental noise, nonlinear response of projector and camera and other unfavorable factors cause jump errors. Taking the actual measurement of resin portraits as an example, as shown in Figure 2, from the analysis of Eq. (5), it can be seen that when the fractional part of the fringe orders $\Delta m_1$ and $\Delta m_2$ are close or $\Delta m_2$ and $\Delta m_3$ are close, a very small error causes $\Delta m_{12}$ and $\Delta m_{23}$ to produce a jump error of 0 to 1, Eq. (7) computes $\Delta m_{123}$ in the same way. From the analysis of Eq. (8), it can be seen that when $\Delta m_{123}$ appears jump error, the error is amplified due to $T_{23}/(T_{23}-T_{12})>1$, so that the solution of $m_{12}$ is incorrect, and then a jump from 0 to 1 will be generated when the integer part $M_{12}$ of the fringe orders is derived in reverse, as shown in Figure 2(a). From the analysis of Eq. (9), it can be seen that when $M_{12}$ has a jump error, due to $T_2/(T_2-T_1)>1$, $M_1$ will also generate a large jump error, as shown in Figure 2(b), and finally can cause $\phi_1$ to generate $n\times 2\pi$ times error, as shown in Figure 2(c), except for the shadow area, the absolute phase has a large error in the edge contour of the measured object, and there are also some error points on the surface of the measured object.

## 3. Phase Unwrapped Method for Modified Fringe Orders

In order to solve the above problems, based on the above analysis of the phase error generation and transmission process, this paper proposes a phase unwrapped method to modify the fringe orders, and the phase information of the fringe with multiple periods is used to solve the absolute phase, that improve the precision of the phase unwrapped on the surface of the measured object, and the unwrapped absolute phase is smooth and has no jump error. The implementation of this method is as follows:

1) Projecting the sinusoidal fringe images with periods of $T_1$, $T_2$ and $T_3$ on the surface of the measured object respectively, and capture the modulated fringe images by the camera;
2) Using the standard four-step phase shift method to solve the wrapped phases $\varphi_1$, $\varphi_2$ and $\varphi_3$ corresponding to $T_1$, $T_2$ and $T_3$;
3) Solving the wrapped phase $\varphi_{12}$ of fringe 12 by the wrapped phase heterodyne of fringe 1 and fringe 2, and solving the wrapped phase $\varphi_{23}$ of fringe 23 by the wrapped phase heterodyne of fringe 2 and fringe 3;
4) Solving the wrapped phase $\varphi_{123}$ of fringe 123 by the wrapped phase heterodyne of fringe 12 and fringe 23, due to $M_{123}=0$, therefore, the wrapped phase of the fringe 123 is the continuous phase $\phi_{123}$;
5) Solving the integer parts $M_{12}$ and $M_{23}$ of the fringe orders. Since the fringe order $m_i\left(i=12,13\right)$ is solved according to Eq. (8) and rounded, the step of the integer part $M_i\left(i=12,13\right)$ of the fringe orders is aligned with the truncation of wrapped phase $\varphi_i\left(i=12,13\right)$, the integer part $M_i\left(i=12,13\right)$ of the fringe orders corresponding to each pixel in the pixel range $\left[nW/T_i,(n+1)W/T_i\right]$ should be the same and differ from the integer part of the fringe orders corresponding to the next pixel area $\left[(n+1)W/T_i,(n+2)W/T_i\right]$ by 1, where $W$ is the width of the captured fringe image, $i=12,13$, $n=\left[0,1,\cdots,(T_i-1)\right]$, so $M_{12}$ and $M_{23}$ can be judged by the front and rear neighborhood points according to Eq. (10), that is,

If

$$\begin{aligned}&\left|M_{12}(x,y+1)-M_{12}(x,y)\right|\geq 1\\&\left|M_{12}(x,y+1)-M_{12}(x,y+2)\right|\geq 1\end{aligned}\tag{10}$$

so

$$M_{12}(x,y+1)=M_{12}(x,y)\tag{11}$$

The calculation of $M_{23}$ is the same, and then use the modified $M_{12}$ and $M_{23}$ to solve the intermediate absolute phases $\phi_{12}$ and $\phi_{23}$.

6) According to the Eq. (12), the integer part $M_2$ of the fringe orders is solved, that is, the average value of the intermediate absolute phases $\phi_{12}$ and $\phi_{23}$ is taken. Then judge the front and rear neighborhood points of $M_2$, and then use the modified $M_2$ to solve the absolute phase $\phi_2$ of the fringe 2 according to Eq. (9).

$$M_2=round[(\frac{T_{12}\times\phi_{12}+T_{23}\times\phi_{23}}{2T_2}-\varphi_2)/2\pi]\tag{12}$$

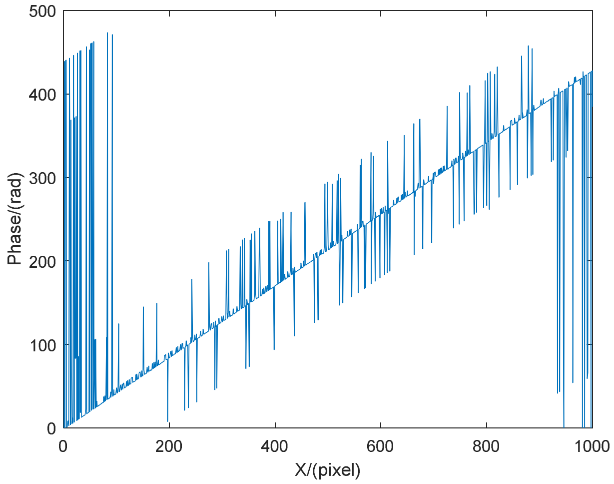
## 4. Simulation Analysis and Experiment

### 4.1. Simulation

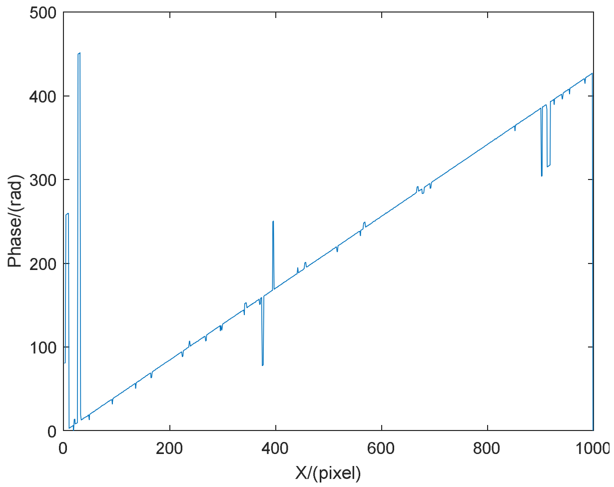
In order to verify the ability of the method of this paper to suppress random noise, Matlab was used to generate a standard sinusoidal fringe images with a resolution of 1024pixel and 768pixel with fringe periods of 70, 64, and 59 [15] respectively. Gaussian noise with mean of 0 dB and variance of 0.01 dB is added to the fringe, and use the method of this paper to solve the phase. As can be seen from Figure 3, the absolute phase obtained by the method of this paper has greatly reduced phase jump error and stronger noise suppression ability compared with the traditional method. In addition, according to the simulation results, when the fringe image is actually projected, it is necessary to avoid using the fringe area at both ends, and use the fringe area in the middle for projection.

***Table 1.*** *RMS phase error of two phase unwrapped methods.*

| Noise (dB) | The traditional method (rad) | the method of this paper (rad) |
|---|---|---|
| 0.01 | 0.1977 | 0.1553 |
| 0.02 | 0.2322 | 0.1740 |
| 0.04 | 0.2693 | 0.2087 |




(a) Phase unwrapped of the traditional method



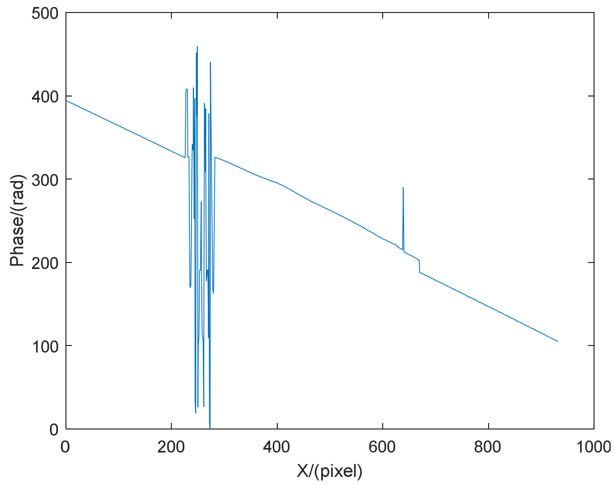

(b) Phase unwrapped of the method of this paper

***Figure 3.*** *Phase unwrapped Comparison.*

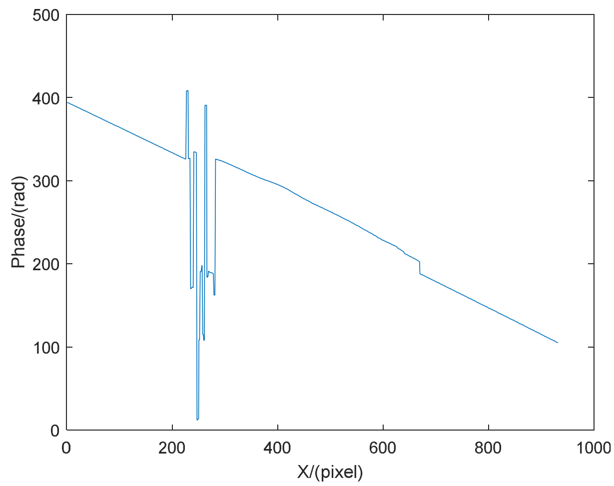
To further compare the effect of phase unwrapped, the phase unwrapped errors of the two methods are compared under different noise intensities, and the root mean square (RMS) error is used as the evaluation criterion. The comparison of the RMS error of the reconstruction errors of the two algorithms is shown in Table 1. It can be seen that the RMS error of the method of this paper is smaller than the traditional three-frequency heterodyne method under the same noise intensity. The RMS phase error is reduced by 23% on average.

### *4.2. Experiment*

To further evaluate the feasibility of the method of this paper, a projector (model: DLP LightCraft4500, resolution: 1280*800) was used to project a standard fringe onto the surface of the measured object; a single camera (model: BFS-U3-13Y3M-C, resolution: 1280*1024) was used for shooting. The traditional three-frequency heterodyne method and the method of this paper are used to measure objects of resin portrait with complex surfaces and wind turbine blade with hole.



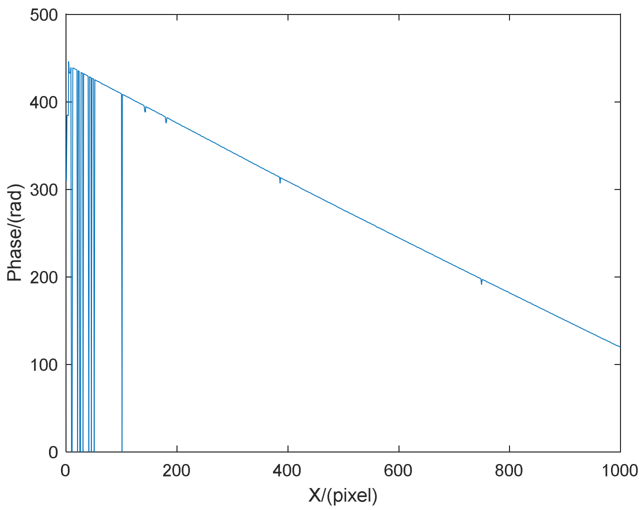

(a) Absolute phase at line 400 of traditional method



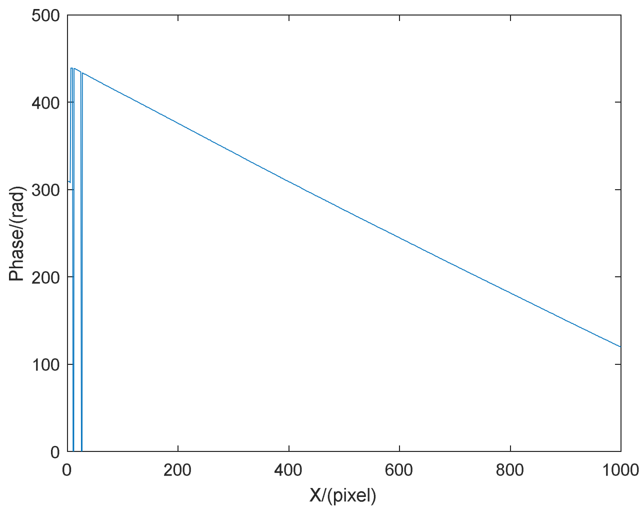

(b) Absolute phase at line 400 of the method of this paper

***Figure 4.*** *Comparison of phase unwrapped results of resin portrait.*

Using a projector to project standard fringes with periods of 70, 64, and 59 on the resin portrait and wind turbine blade, and to use the standard four-step phase shift method to solve the wrapped phase and use the traditional three-frequency heterodyne method and the method of this paper to expand the wrapped phase respectively, taking the 400th row as an example, as shown in Figure 4(a) and Figure 5(a) are the absolute phase of the traditional method, and Figure 4(b) and Figure 5(b) are the absolute phase of the method of this paper, the absolute phase obtained by the traditional method has a large jump error, and the absolute phase obtained by the method of this paper is relatively smooth, and has a strong inhibitory effect on the phase jump error. As can be seen from the phase mesh in Figure 6, the method of this paper can also unwrap the phases of complex surfaces and hole defect well and has a strong ability to distinguish the details of the measured object.

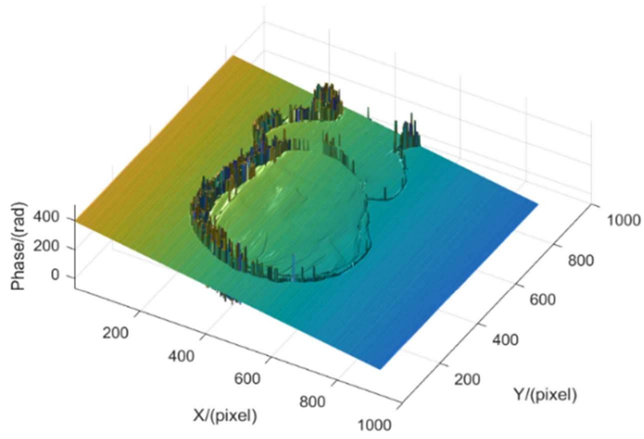



(a) Absolute phase at line 400 of traditional method

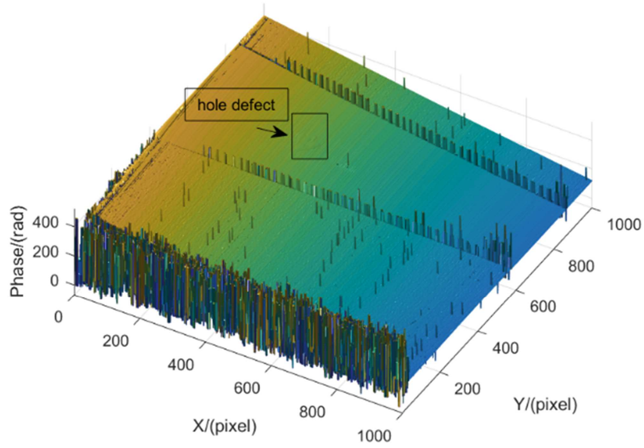



(b) Absolute phase at line 400 of the method of this paper

***Figure 5.*** *Comparison of phase unwrapped results of wind turbine blade.*




(a) Resin portrait




(b) Wind turbine blade

***Figure 6.*** *The mesh of absolute phase.*

# 5. Conclusion

Due to factors such as environmental noise and the nonlinear response of the projector during the actual measurement, there is an error in the wrapped phase. In the process of using the three-frequency heterodyne method to solve the absolute phase, the error will be transmitted and amplified, affecting the phase unwrapped accuracy. Therefore, A phase unwrapped method for modifying fringe orders is proposed, which eliminates the jump error in the process of phase unwrapped and improves the three-dimensional detection accuracy. The method of this paper uses the neighborhood points to modify the fringe orders, and base on the different recognition ability of the fringes with different periods, the phase information of fringes with multiple periods is used to solve the absolute phase of fringe with intermediate period, which improves the phase unwrapped accuracy and is conducive to the subsequent improvement of 3D detection accuracy. Both simulation and experimental results show that, compared with the traditional three-frequency heterodyne method, the method of this paper has a good correction effect on the phase jump error, and the phase error is reduced by an average of 23%. This paper focuses on the improvement of phase accuracy in multi-frequency heterodyne method, but in order to adapt to various working conditions in the actual industrial environment, we can pay attention to the application of multi-frequency heterodyne method in the measurement of reflective or dark objects.

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